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Streamlines in 2-Dimensional Vector Fields

Introduction

This worksheet gives two examples of Maple's capabilities for calculating and displaying the streamlines in a 2-dimensional vector field.

Example 1

A vector field **v** is defined by: $v(x, y) = v_x i + v_y j$. The streamlines show the path traced by an object following the vector field and can be plotted by defining the first order ordinary differential equation: $\frac{dy}{dx} = \frac{v_y}{v_x}$. In this example, a vector field **v** is defined by $v(x, y) = (x + y)\, i + (x - y)\, j$

$$\frac{\partial}{\partial x}\, y(x) = \frac{x - y(x)}{x + y(x)}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\, y(x) = \frac{x - y(x)}{x + y(x)}$$

$dsolve(\mathbf{(2.1)}, y(x))$

$$y(x) = \frac{-x\,\_C1 - \sqrt{2\,x^2\,\_C1^2 + 1}}{\_C1},\; y(x) = \frac{-x\,\_C1 + \sqrt{2\,x^2\,\_C1^2 + 1}}{\_C1} \qquad \mathbf{(2.2)}$$

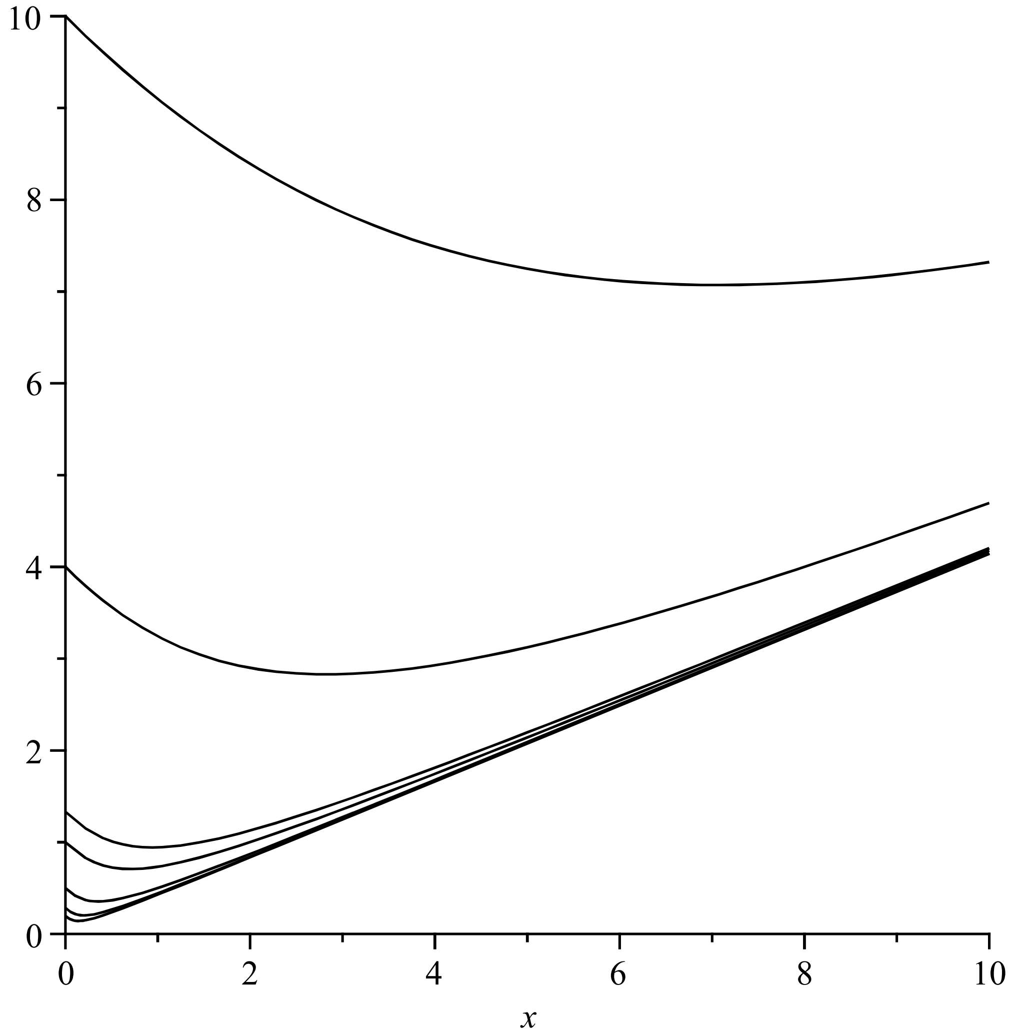
As explicit solutions:

$exp1 := solve(\mathbf{(2.2)}[1], y(x)), solve(\mathbf{(2.2)}[2], y(x))$

$$-\frac{x\,\_C1 + \sqrt{2\,x^2\,\_C1^2 + 1}}{\_C1},\; \frac{-x\,\_C1 + \sqrt{2\,x^2\,\_C1^2 + 1}}{\_C1} \qquad \mathbf{(2.3)}$$

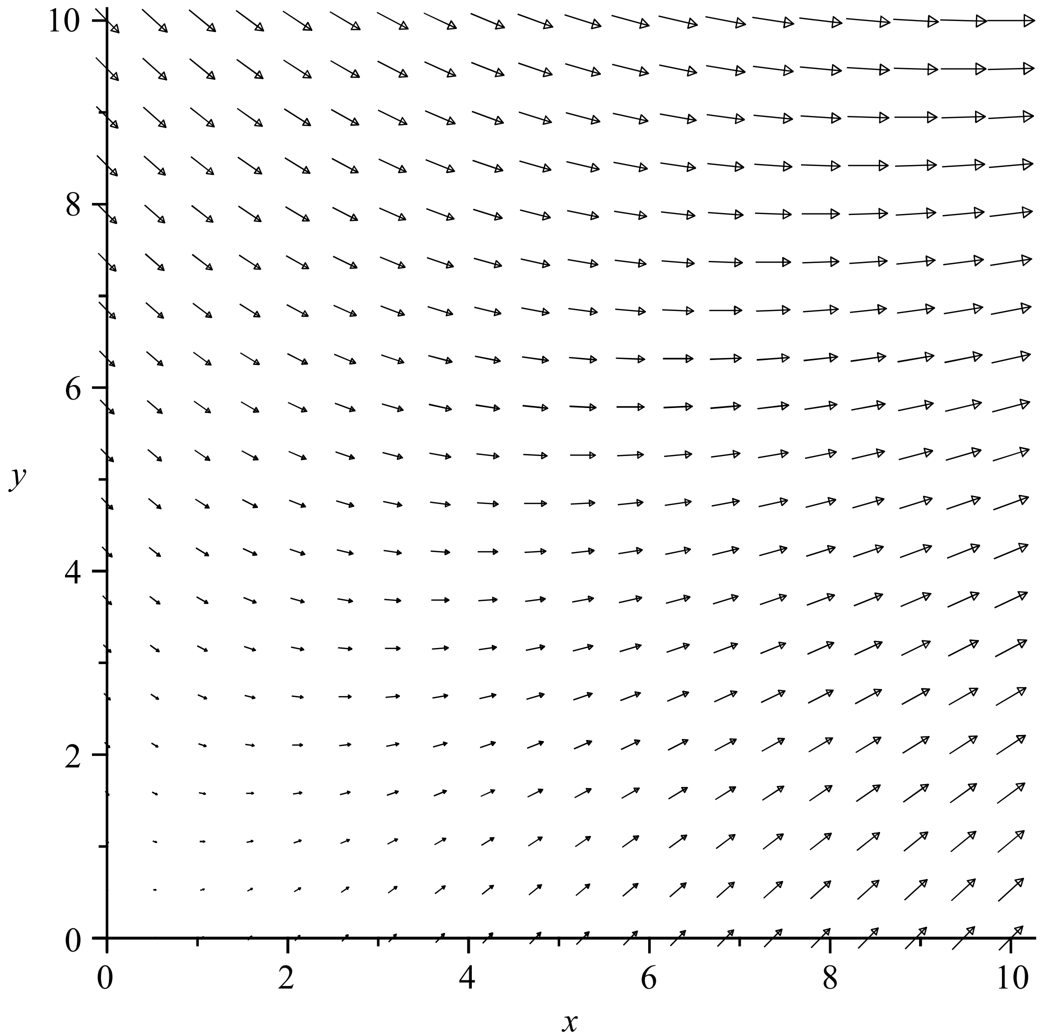
The family of solutions is now plotted in the first quadrant to display the sections of streamlines for various values of _C1:

```
plot( { subs(_C1 =-0.1, exp1[1]), subs(_C1 =-.25, exp1[1]),
    subs(_C1 =-5, exp1[1]), subs(_C1 =-.75, exp1[1]),
    subs(_C1 =-1, exp1[1]), subs(_C1 =-2, exp1[1]),
    subs(_C1 =-3.5, exp1[1]), subs(_C1 =-5, exp1[1])
    }, x = 0 ..10, 0 ..10, color = black);
```



Alternatively samples of the vector field can be plotted directly:

with(*plots*) :
fieldplot($[x + y, x - y]$, $x = 0..10$, $y = 0..10$, *arrows* = *slim*);



Comparing these two plots, the solutions of the original differential equation indicate the direction of the vector field at any point but to see the magnitude and direction of the field, the fieldplot procedure must be used.

Example 2

Repeat the above analysis for the vector field: $v = e^{(-x)} \cdot \cos(x) i - e^{(-x)} \cdot \sin(x) j$. The differential equation defining the streamlines is:

$\frac{\partial}{\partial x} y(x) = -\tan(x)$

$$\frac{d}{dx} y(x) = -\tan(x) \qquad \textbf{(3.1)}$$

whose general solution is:

dsolve(**(3.1)**)

$$y(x) = \ln(\cos(x)) + \_C1 \quad (3.2)$$

exp2 := *solve*(**(3.2)**, *y*(*x*))

$$\ln(\cos(x)) + \_C1 \quad (3.3)$$

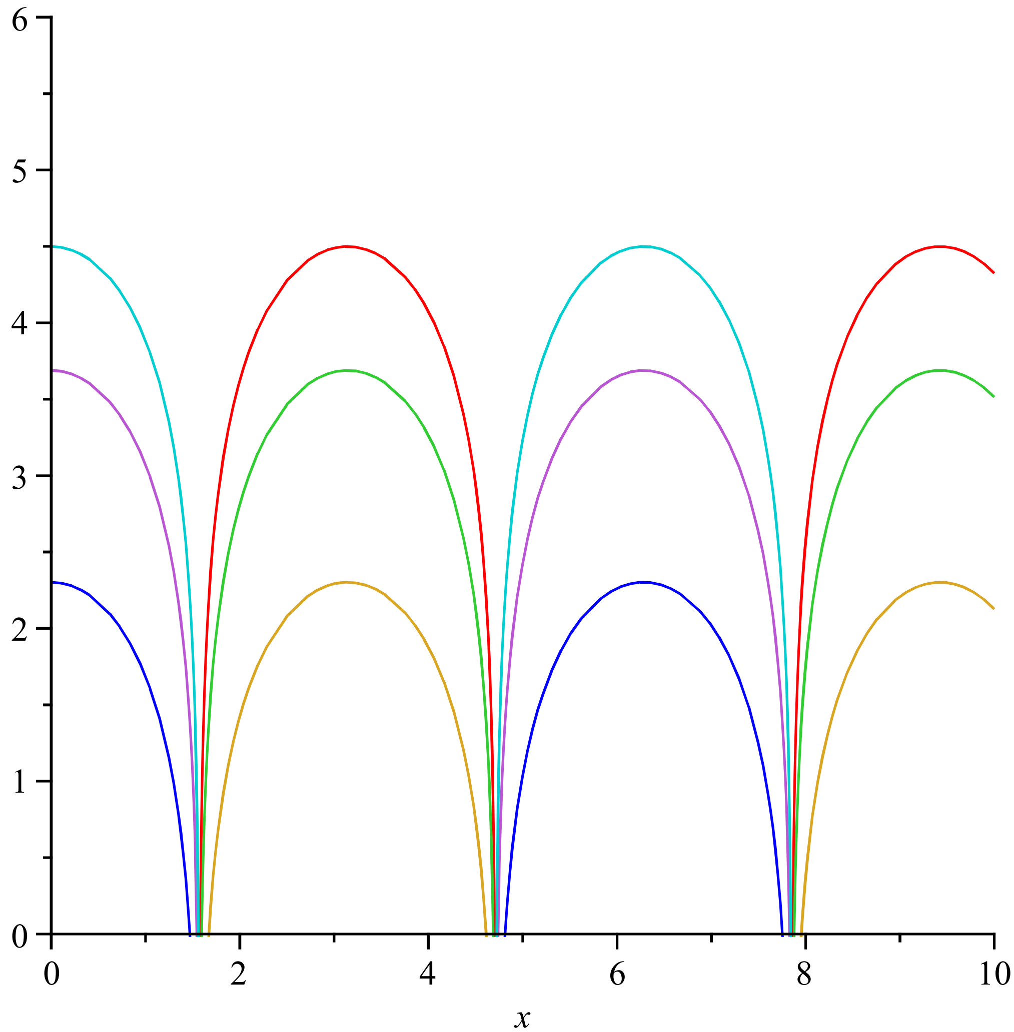
The expression may be written as below without consequence:

exp2 := ln(_*C1* * cos(*x*))

$$\ln(\_C1 \cos(x)) \quad (3.4)$$

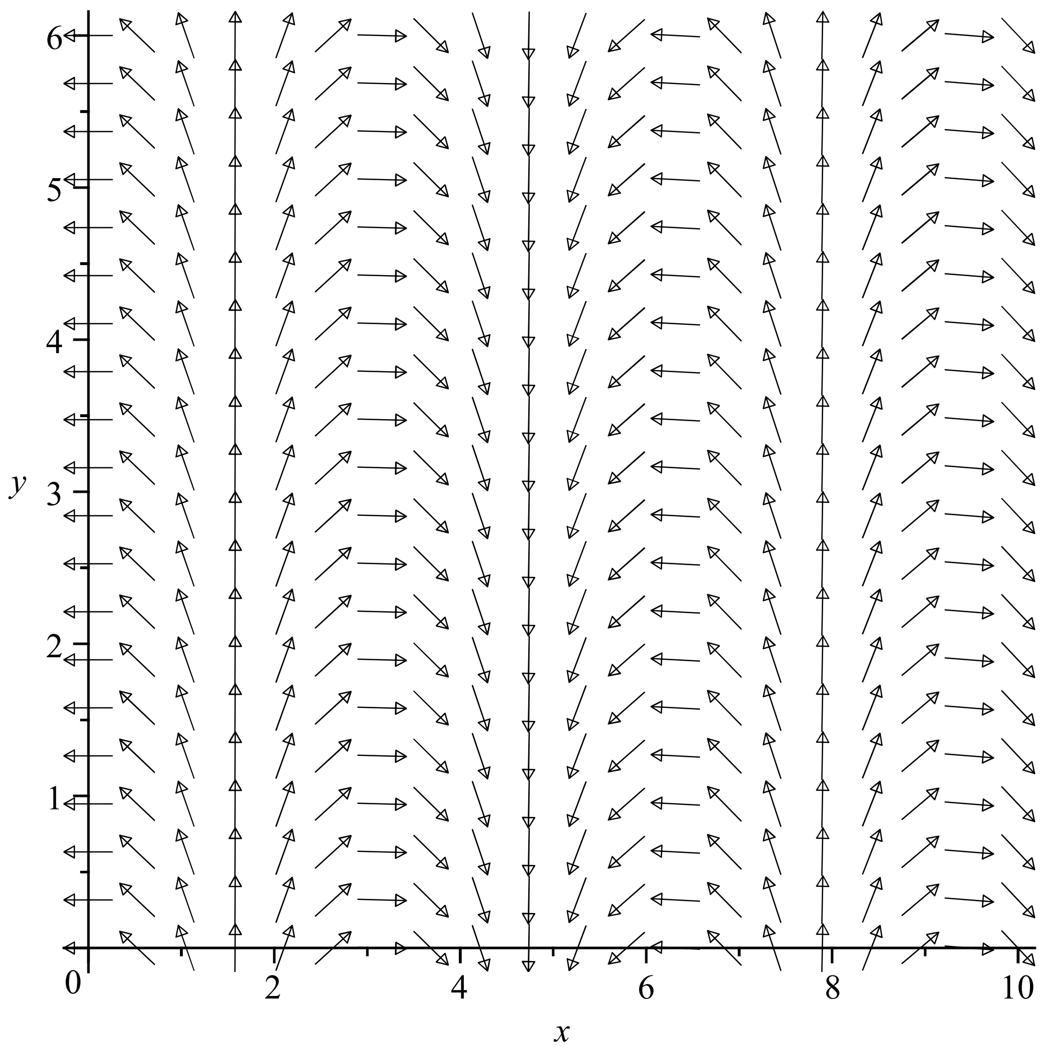
Plotting the family of curves for values of _C1 gives:

```
plot( { subs(_C1 =-90, exp2), subs(_C1 = 90, exp2),
     subs(_C1 = 10, exp2), subs(_C1 =-10, exp2),
     subs(_C1 = 40, exp2), subs(_C1 =-40, exp2)
    }, x = 0 ..10, 0 ..6, style = LINE);
```



Plotting the field components directly gives:

```
with(plots) :
fieldplot( [-cos(x), sin(x)], x = 0 ..10, y = 0 ..6, arrows = SLIM);
```



y
x
0
2
4
6
8
10
1
2
3
4
5
6