2nd Order DE Solution Examples

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Process	Equation 1: Homogeneous	Equation 2: Non Homogeneous
Define the equation and various sets of initial conditions.	$eq1 := 36\cdot diff(y(t),t,t) - 12\cdot diff(y(t),t) + 37\cdot y(t) = 0$ $36\left(\frac{d^2}{dt^2}y(t)\right) - 12\left(\frac{d}{dt}y(t)\right) + 37\,y(t) = 0$ **(1)** $ics := y(0)=0, D(y)(0)=v0$ $y(0)=0, D(y)(0)=v0$ **(2)** $ics2 := y(0)=0, D(y)(0)=.2$ $y(0)=0, D(y)(0)=0.2$ **(3)** $ics3 := y(0)=0, D(y)(0)=.4$ $y(0)=0, D(y)(0)=0.4$ **(4)** $ics4 := y(0)=0, D(y)(0)=0.8$ $y(0)=0, D(y)(0)=0.8$ **(5)**	$eq2 := 36\cdot diff(y(t),t,t) - 12\cdot diff(y(t),t) + 37\cdot y(t) = t^2$ $36\left(\frac{d^2}{dt^2}y(t)\right) - 12\left(\frac{d}{dt}y(t)\right) + 37\,y(t) = t^2$ **(6)**
Solve the equation both generally, and for each of the sets of initial conditions.	$sol := rhs(dsolve(\{eq1, ics\}, y(t)))$ $v0\,e^{\frac{1}{6}t}\sin(t)$ **(7)** $yp := \frac{d}{dt}$ **(7)** $\frac{1}{6}v0\,e^{\frac{1}{6}t}\sin(t) + v0\,e^{\frac{1}{6}t}\cos(t)$ **(8)**	$sol5 := rhs(dsolve(\{eq2, ics\}, y(t)))$ $\left(v0 - \frac{1284}{50653}\right)e^{\frac{1}{6}t}\sin(t) + \frac{2376}{50653}e^{\frac{1}{6}t}\cos(t) + \frac{1}{37}t^2 + \frac{24}{1369}t - \frac{2376}{50653}$ **(15)** $sol6 := subs(\_C2 = 1,$

$sol2 := rhs(dsolve(\{eq1, ics2\}, y(t)))$

$$\frac{1}{5}\, e^{\frac{1}{6} t} \sin(t) \quad \textbf{(9)}$$

$yp2 := \frac{d}{dt}$ **(9)**

$$\frac{1}{30}\, e^{\frac{1}{6} t} \sin(t) + \frac{1}{5}\, e^{\frac{1}{6} t} \cos(t) \quad \textbf{(10)}$$

$sol3 := rhs(dsolve(\{eq1, ics3\}, y(t)))$

$$\frac{2}{5}\, e^{\frac{1}{6} t} \sin(t) \quad \textbf{(11)}$$

$yp3 := \frac{d}{dt}$ **(11)**

$$\frac{1}{15}\, e^{\frac{1}{6} t} \sin(t) + \frac{2}{5}\, e^{\frac{1}{6} t} \cos(t) \quad \textbf{(12)}$$

$sol4 := rhs(dsolve(\{eq1, ics4\}, y(t)))$

$$\frac{4}{5}\, e^{\frac{1}{6} t} \sin(t) \quad \textbf{(13)}$$

$yp4 := \frac{d}{dt}$ **(13)**

$$\frac{2}{15}\, e^{\frac{1}{6} t} \sin(t) + \frac{4}{5}\, e^{\frac{1}{6} t} \cos(t) \quad \textbf{(14)}$$

$rhs(dsolve(\{eq2, ics2\}, y(t))))$

$$\frac{44233}{253265}\, e^{\frac{1}{6} t} \sin(t) + \frac{2376}{50653}\, e^{\frac{1}{6} t} \cos(t) + \frac{1}{37}\, t^2 + \frac{24}{1369}\, t - \frac{2376}{50653} \quad \textbf{(16)}$$

$sol7 := subs(\_C2 = 1, rhs(dsolve(\{eq2, ics3\}, y(t))))$

$$\frac{94886}{253265}\, e^{\frac{1}{6} t} \sin(t) + \frac{2376}{50653}\, e^{\frac{1}{6} t} \cos(t) + \frac{1}{37}\, t^2 + \frac{24}{1369}\, t - \frac{2376}{50653} \quad \textbf{(17)}$$

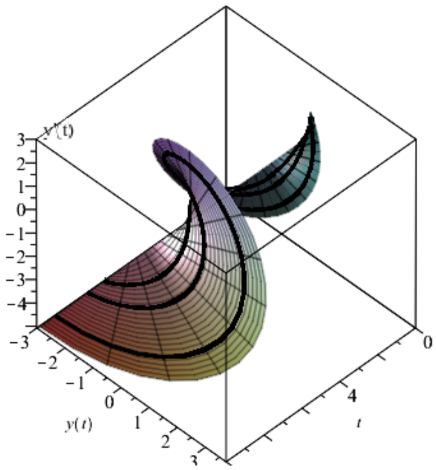
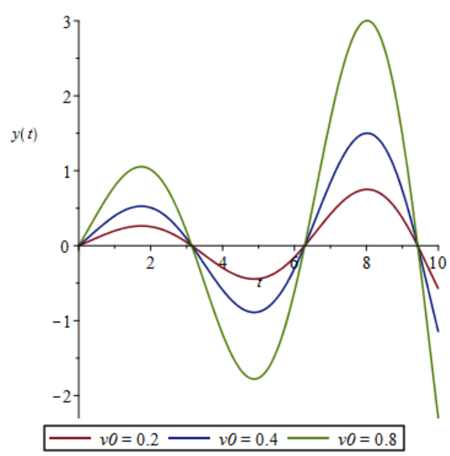
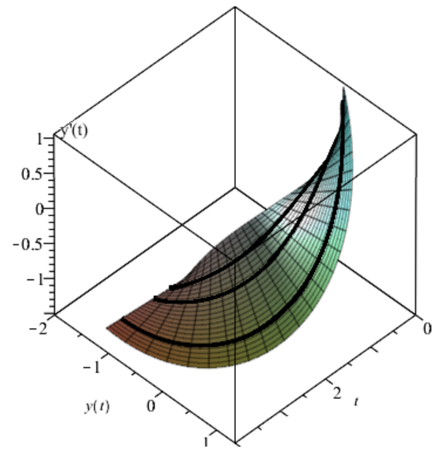
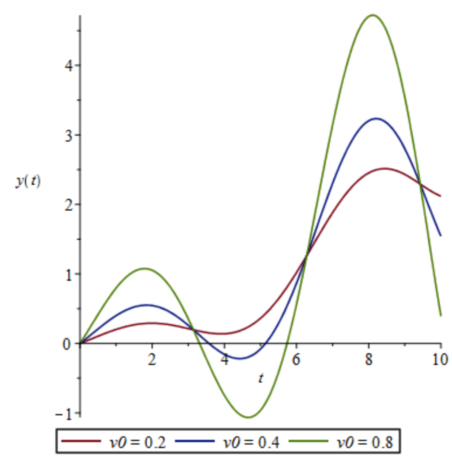
$sol8 := subs(\_C2 = 1, rhs(dsolve(\{eq2, ics4\}, y(t))))$

$$\frac{196192}{253265}\, e^{\frac{1}{6} t} \sin(t) + \frac{2376}{50653}\, e^{\frac{1}{6} t} \cos(t) + \frac{1}{37}\, t^2 + \frac{24}{1369}\, t - \frac{2376}{50653} \quad \textbf{(18)}$$

$yp5 := \frac{d}{dt}\, sol5$

$$\frac{1}{6}\left(v0 - \frac{1284}{50653}\right) e^{\frac{1}{6} t} \sin(t) + \left(v0 - \frac{1284}{50653}\right) e^{\frac{1}{6} t} \cos(t) \quad \textbf{(19)}$$

		$+\frac{396}{50653}\mathrm{e}^{\frac{1}{6}t}\cos(t)$ $-\frac{2376}{50653}\mathrm{e}^{\frac{1}{6}t}\sin(t)$ $+\frac{2}{37}t+\frac{24}{1369}$ $yp6 := \frac{\mathrm{d}}{\mathrm{d}t}sol6$ $-\frac{731}{41070}\mathrm{e}^{\frac{1}{6}t}\sin(t)$ **(20)** $+\frac{1249}{6845}\mathrm{e}^{\frac{1}{6}t}\cos(t)$ $+\frac{2}{37}t+\frac{24}{1369}$ $yp7 := \frac{\mathrm{d}}{\mathrm{d}t}sol7$ $\frac{319}{20535}\mathrm{e}^{\frac{1}{6}t}\sin(t)$ **(21)** $+\frac{2618}{6845}\mathrm{e}^{\frac{1}{6}t}\cos(t)$ $+\frac{2}{37}t+\frac{24}{1369}$ $yp8 := \frac{\mathrm{d}}{\mathrm{d}t}sol8$ $\frac{1688}{20535}\mathrm{e}^{\frac{1}{6}t}\sin(t)$ **(22)** $+\frac{5356}{6845}\mathrm{e}^{\frac{1}{6}t}\cos(t)$ $+\frac{2}{37}t+\frac{24}{1369}$
Plot the solution. The surface shows the solution to the DE found above, using y'(0)=v0 as an initial condition. The thick black lines through the surface show 3 different cases, where we	$plot1 := plot3d([t, sol, yp], t = 0..10, v0 = 0..1, labels = [t, y(t), ""]):$ $plot2 := plot3d([t, sol2, yp2], t = 0..10, v0 = 0.2..0.2, thickness = 4):$ $plot3 := plot3d([t, sol3, yp3], t = 0..10, v0 = 0.4..0.4, thickness = 4)$	$plot5 := plot3d([t, sol5, yp5], t = 0..4, v0 = 0..1, labels = [t, y(t), ""]):$ $plot6 := plot3d([t, sol6, yp6], t = 0..4, v0 = 0.2..0.2, thickness = 4):$ $plot7 := plot3d([t, sol7, yp7], t = 0..4, v0 = 0.4..0.4, thickness = 4):$ $plot8 := plot3d([t, sol8, yp8], t = 0..4, v0 = 0.8..0.8, thickness = 4):$ $plots[display]([plot5, plot6, plot7,$

<table>
<tr>
<td>have chosen a specific value for v0.</td>
<td>:
plot4 := plot3d([t, sol4, yp4], t = 0 ..10, v0 = 0.8 ..0.8, thickness = 4)
:
plots[display]([plot1, plot2, plot3, plot4, plots[textplot3d]([10, −3, 3,"y'(t)"], align = {above, right})])

plot([sol2, sol3, sol4], t = 0 ..10, labels = [t, y(t)], legend = [v0 = 0.2, v0 = 0.4, v0 = 0.8])
</td>
<td>plot8, plots[textplot3d]([4, −2, 1, "y'(t)"], align = {above, right})])

plot([sol6, sol7, sol8], t = 0 ..10, labels = [t, y(t)], legend = [v0 = 0.2, v0 = 0.4, v0 = 0.8])
</td>
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